# MATHEMATICAL MODEL OF COLUMN DRYING

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Differential equations are proposed to describe the drying of a finely disperse homogeneous material in a pneumatic drying column. A method of numerical solution is suggested.

One method of intensifying the drying process is to use a fluidized bed (boiling, pneumotransport, swirling flow, etc.). Most characteristically, drying under fluidized conditions involves pneumatic drying columns, which are simple in construction, provide high intensity (the moisture output reaches 1000 $kg/m^3 \cdot h$), and allow a high heat-carrier temperature to be used even when the material to be dried is thermally unstable.

Because the mechanisms of the processes occurring in dryer columns are complex, analytic calculation is difficult and therefore it is usual to carry out an experimental investigation, making use of similarity theory.

Column drying is generally described by the equations of convective heat transfer [1-3]. The heat-transfer coefficients are assumed to be constant over the length of the column, but this simplification considerably distorts the true picture of the process. Change in the heat-transfer coefficients is taken into account in the methods of calculation for heat-transfer apparatus of fluidized-bed type outlined in [4]. A mathematical model of column drying is given in [5], but heat transfer between the particles and the gas is ignored. Systems of equations of heat and mass transfer describing drying may be solved, taking into account change in the transfer coefficients, by means of a computer [6].

The present paper outlines a mathematical model of drying in a pneumatic drying column. It is assumed that the wet material is monodisperse and that all the parameters of the process are constant over the column cross section (one-dimensional model). The drying is a steady process. The temperature and moisture content of the particles are constant over the cross section. Because of the high velocity of the drying agent (the gas), the convective heat and mass flows along the column are considerable in comparison with the diffusional flows, and hence diffusion of the vapor in the gas flow and heat conduction of the gas over the length of the tube may be neglected.

## Kinetic Equations of Mass and Heat Transfer

At a distance x from the column inlet, an element of volume of height dx and base of unit area is isolated. Evaporation of moisture from the material in this volume to the gas $F(v_M)\beta[P(T) - p]dx$ leads to a change in the amount of moisture in the material $-d[G_M(W)W]$.

Thus, the mass-transfer equation may be written in the form

$$\frac{d}{dx}[G_M(W)W] + F(v_M)\beta[P(T) - p] = 0. \tag{1}$$

In the volume dx the heat loss of the gas is $F(v_M)\alpha(\theta - T)dx$; this heat produces a change in the enthalpy of the material $d[G_M(W)C_M(W)T]$ and evaporation of moisture from the material into the gas $F(v_M)\beta[P(T) - p]r_T dx$.

The heat transfer between the drying agent (the gas) and the material is described by the equation

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$$\frac{d}{dx}[G_M(W)C_M(W)T]+F(v_M)\{\alpha(T-\theta)+\beta[P(T)-p]r_T\}=0. \tag{2}$$

## Equation of Motion of Particles of Material

Let a particle of diameter $\delta$ and mass

$$m=\frac{\pi\delta^3}{6}\rho_M(W)$$

be moving with velocity $v_M$ at time t. Its momentum is $q = mv_M$. In the interval of time dt, mass dm leaves the particle with velocity u. At time t + dt, the velocity of the particle is $v_M + dv_M$ and the momentum is

$$q'=(m-dm)(v_M+dv_M)+udm.$$

By Newton's second law

$$q'-q=Fdt,$$

where $F = F_1 + F_2$ is the net force acting on the particle; $F_1 = -mg$ is the weight of the particle; $F_2 = \Phi(\overrightarrow{v_g - v_M})$ is the aerodynamic drag of the medium in the direction of the vector $\overrightarrow{v_g - v_M}$, with numerical value

$$\xi\frac{\pi\delta^2}{8}\rho(\theta,\ p)(v_g-v_M)^2.$$

In view of the form of q and q',

$$m\frac{dv_M}{dt}-v_M\frac{dm}{dt}+u\frac{dm}{dt}=F.$$

In this equation it is possible to set $u - v_M$. Since $dx/dt = v_M$, taking into account the form of F gives finally the following equation of motion for a particle of the wet material:

$$v_M\frac{dv_M}{dx}+g-\frac{1}{m}\Phi(\overrightarrow{v_g-v_M})=0. \tag{3}$$

## Equations of Moisture and Heat Balance

If $G_g$ is the mass flow rate of dry gas through a cross section of the column in unit time, and d is the moisture content of the gas, the balance equation for moisture is

$$G_M(W)W+G_gd=G_M(W_i)W_i+G_gd_i. \tag{4}$$

The subscript i denotes the initial (inlet) value in the drying column.

Let $Q_M$ denote the heat leaving the wet material, $Q_g$ the heat leaving the gas, $Q_{evap}$ the heat necessary for evaporation of the moisture, and $Q_l$ the heat loss through the wall between the inlet of the column and the cross section under consideration; then the heat-balance equation at any point x of the drying column is

$$Q_M+Q_g+Q_{evap}+Q_l=Q, \tag{5}$$

where

$$Q_l=\frac{\Lambda}{S}K\int_0^x[\theta(\xi)-T_{ext}]d\xi;$$

Q is the heat supplied to the drying column by the gas and the wet material.

The system (1)-(5) describes the drying of a monodisperse material in a pneumatic drying column; it can be solved if the following initial conditions are imposed:

$$W|_{x=0}=W_i,\ T|_{x=0}=T_i,\ v_M|_{x=0}=v_{M,i},\ d|_{x=0}=d_i,\ \theta|_{x=0}=\theta_i. \tag{6}$$

The system of linear equations (1)-(5) will be solved by a numerical method.

## Solution of Eqs. (1)-(5)

To solve Eqs. (1)-(5), the whole length of the drying column L is divided into N equal parts by the points $x_j$, where j = 0, 1, ..., N ($x_0 = 0$, $x_N = L$). Euler's method [7] is used

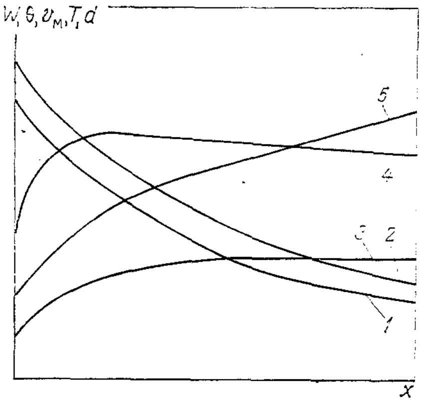


Fig. 1. Variation of parameters along column length: 1) $W = W(x)$; 2) $\theta = \theta(x)$; 3) $v_M = v_M(x)$; 4) $T = T(x)$; 5) $d = d(x)$.

to find the values of W, T, $v_M$, θ, and d at the grid points $x_j$. This involves replacing the initial derivatives in Eqs. (1)-(3) by difference relations and passing from Eqs. (1)-(5) to a system of nonlinear algebraic equations. For example, Eq. (1) is replaced by the equation

$$\frac{G_M(W_{j+1})W_{j+1} - G_M(W_j)W_j}{h} + F(v_{M,j})\beta_j[P(T_j) - p_j] = 0. \tag{7}$$

here $h = L/N$ and the subscript j + 1 or j denotes the value taken at the point $x_{j+1}$ or $x_j$.

The resulting system is solved successively, from point to point, using iteration at each step. Thus, if the values of all the functions are known at $x_j$, $W^1_{j+1}$ can be found from Eq. (7) and $T^1_{j+1}$, $v^1_{M,j+1}$, $d^1_{j+1}$, and $\theta^1_{j+1}$ from the other equations of the system (not given here). Taking these values instead of the values at $x_j$, the process is repeated to give $W^2_{j+1}$, $v^2_{M,j+1}$, $T^2_{j+1}$, $d^2_{j+1}$, and $\theta^2_{j+1}$. Since only two successive iterations are required to give the desired accuracy, the calculation is stopped here, and the resulting values are taken as the values of the solution at the point $x_{j+1}$. Repeating this process gives the solution at the point $x_{j+2}$ and so on.

Thus, the distributions of all the parameters of the drying process over the length of the drying column are found. It is easily established that as $h \to 0$ the solution obtained in this way differs from the accurate solutions of Eqs. (1)-(5) by an amount of order h.

## Results of Numerical Experiment

Computer calculations of the drying of sodium fluorosilicate were carried out to verify the solutions of Eqs. (1)-(5) from given initial data. The calculations showed that the final moisture content of the material decreased with decrease in the diameter of the wet particles, increase in the initial gas velocity, decrease in the input of material to the drying column, and increase in the initial temperature of the gas.

Results of one of the calculations are shown in Fig. 1; the removal of moisture from the material evidently occurs mainly in the initial portion of the column. This may be attributed to the high relative velocity of the gas and the wet material, and hence the large coefficients of heat and mass transfer, and to the high gas temperature in this region.

The results of these numerical calculations agree with experimental data [1-3, 5, 8].

The results of computer calculation of the drying of sunflower seeds also agree satisfactorily with experimental data. For example, the final moisture content of the material was 12.75% by experiment and 12.67% by numerical calculation; the final temperatures of the wet material and the gas were 61.2 and 340°C by experiment, and 70 and 360°C by calculation.

Comparison of numerical calculations with experimental data shows that the mathematical model of drying in Eqs. (1)-(5) satisfactorily describes the drying of a monodisperse material in a pneumatic drying column.

## NOTATION

$G_M(W)$, mass flow rate of material with moisture content W through cross section x in unit time; $F(v_M)$, surface of wet material included in column volume of unit length (depends on velocity of material in column, $v_M$); P(T), vapor tension above surface of material at temperature T; p, partial pressure of water vapor in gas; $\alpha$, $\beta$, coefficients of heat and mass transfer; $\theta$, gas temperature; $C_M(W)$, enthalpy of material at moisture content W; $\rho_M(W)$, density of material at moisture content W; $\xi$, drag coefficient of medium; $\rho(\theta, p)$, gas density at temperature $\theta$ and water-vapor partial pressure p; $v_g$, gas velocity; g, acceleration due to gravity; d, moisture content of gas; $r_T$, specific heat of vaporization at temperature T; $\Lambda$, circumference of drying column; K, coefficient of heat transfer from gas through wall to surrounding medium at temperature $T_{ext}$; S, cross-sectional area of drying column.

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# KINETICS OF THE SOLUTION OF POLYOXYETHYLENE IN WATER

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The kinetics of the process of solution of WSR-301 polyoxyethylene in water is investigated using a quadratic Doppler spectrometer.

It is known that several weak polymer solutions have anomalously low drag in turbulent motion along tubes. The most pronounced effect is noted for the synthetic polymer polyoxyethylene (PEO), the hydrodynamic action of which depends on the molecular weight, concentration, and conditions of technical preparation of the sample. It is of interest to study these properties, so as to determine the optimal parameters for efficiency of PEO as a drag-reducing agent.

The present investigation is devoted to WSR-301 PEO, which has the following characteristics. For velocity gradient g = 0, the characteristic viscosity, measured on a Zimm rotational viscometer, is $[\eta]_{g=0} = 6.5$ dl/g. The Mark–Kuhn–Khauvink formula $[\eta]^{25°C}_{water} = 1.25 \cdot 10^4 \cdot M^{0.78}$ gives a value of $1 \cdot 10^6$ for the molecular weight. The maximum reduction in drag is observed approximately 2 h after (as shown by visual observation and decanting the water–polymer mixture) solution of PEO in water is complete. If the solution is kept for some time, a sharp drop in its efficiency is observed within the first day [1]. The aim of the present work is to explain these changes in the drag-reducing properties of PEO.

It seems likely that change in the efficiency of the polymer with time is associated with changes in the structure of the macromolecules in the solution or with the dynamics of the process of solution. A clearer idea can be obtained by studying the light-scattering properties of the solution over various time intervals from its moment of preparation. It is known that light scattered by moving particles undergoes a Doppler frequency shift. For suspended particles diffusing in liquids, this shift may vary from a fraction of a hertz to

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